

FIG. 2. (Color online) An example of a stent.

analyzed the flow pattern formation within the aneurysm in detail, show that our model can provide an effective simulation tool for the stented-blood-flow problem. Moreover compared with a traditional CFD solver, such as fluent, the LB approach is quite flexible and easy to adapt to various new conditions. For instance, in a forthcoming work, we shall consider explicitly the clotting effect, which we would like to describe as a deposition-aggregation process, similar to the LB sediment transport model of Refs. [25,26]. Also, we plan to include the wall elasticity in order to study aneurysm's growth and rupture, using the model described in Ref. [27].

The situation we consider is illustrated in Fig. 1. The size and parameters of the simulation are chosen so as to correspond to the range of experimental and clinical observations [3,8]. The stent is a tubular mesh which fits perfectly with the parent vessel width. In our two-dimensional simulation, the stent is represented by horizontal struts (see Fig. 1) with a thickness of two lattice cells. We use three types of stents *A–C* of different pore sizes and with relatively long periodic designs, which are suitable for measuring the two-dimensional positioning effect. The combination of the pore size and strut size is as follows: stent *A* (15-cell strut, 55-cell pore, 35-cell strut), stent *B* (15-cell strut, 40-cell pore, 35-cell strut), and stent *C* (15-cell strut, 15-cell pore, 35-cell strut), therefore the characteristic cyclic lengths of these stents are 160, 130, and 80 cells, respectively.

As for the aneurysm, we use two different cavities with a wide and a narrow neck in order to consider the influence of the neck size on the flow reduction. We use the following parameters based on the typical clinical observation of a saccular cerebral aneurysm. The aneurysm diameter is 10 mm and occupies 200 lattice cells in our simulation. The orifice diameter of the small-necked aneurysm is 5 mm and that of the large-necked aneurysm is 10 mm. The parent vessel is a straight tube, 4 mm wide (80 cells) and 40 mm long (800 cells).

In the parent vessel which we focus on, we can observe the following clinical flow parameters: the fluid density  $\rho = 1.087 \times 10^3$  kg/m<sup>3</sup>, fluid viscosity  $\mu = 3.695 \times 10^{-3}$  Pa s, the average fluid velocity  $u = 2.92 \times 10^{-1}$  m/s, the mean volume flow  $Q = 2.2 \times 10^{-3}$  m<sup>3</sup>/s, and systolic and diastolic pressure  $p$  of  $1.9326 \times 10^4$  and  $1.1996 \times 10^4$  Pa. We adjust the time and length scales between the numerical simulation and clinical data through the Reynolds number  $Re = \rho U l / \mu$ , where  $U$  and  $l$  are the characteristic velocity and the characteristic length in the system. Here we use the av-

erage velocity  $u = 2.92 \times 10^{-1}$  m/s in the parent vessel for the characteristic velocity  $U$ , and the diameter  $d = 4.0 \times 10^{-3}$  m for the characteristic length  $l$ . Then we get the clinical Reynolds number  $Re \approx 344$  in the parent vessel. For the numerical study, we use the following flow parameters in order to get reasonable Reynolds number with numerical stability and efficiency [20]. Here we use the two-dimensional square lattice with nine velocities for the calculation, which is called the D2Q9 model [14,20]. The average density  $\rho = 1.0$  and the kinematic viscosity of the lattice Boltzmann fluid  $\nu = \mu / \rho = (2\tau - 1) / 6 = 0.026$ , where the relaxation time for the LB algorithm  $\tau = 0.58$ . The velocity at the center of the parent vessel  $u = -(d^2 / 8\mu) \nabla p \approx 0.1$ , which is drawn by the pressure gradient  $|\nabla p| = 4.17 \times 10^{-6}$  between the inlet and the outlet, is chosen as the characteristic velocity  $U$ . In our D2Q9 model the lattice Boltzmann sound speed  $c_s = 1/\sqrt{3}$ , therefore the Mach number  $M = u / c_s \approx 0.173$ . The parent vessel diameter  $d = 80$  is used for the characteristic length  $l$ . As a result we get the Reynolds number  $Re \approx 307$  in the parent vessel.

The main flow obeys periodic boundary conditions along the  $x$  axis defined in Fig. 1, and is driven by a pressure gradient between the inlet and the outlet, which is imposed according to the method described in Ref. [28]. On the walls of the vessel and the aneurysm we use the so-called bounce-back rule [19,28–30], which ensures a nonslip boundary condition. On a stent element, the fluid particles also bounce back. The convergence criterion is attained by comparing simulation results from two successive time steps and the stop criterion is when this difference is less than  $10^{-7}$ .

We show that we can characterize the flow reduction property of the stent in a quantitative way by introducing a measure of the flow reduction using the positioning effect. Here we define the stent positioning effect as the phenomenon caused by the variation of the strut position depending on the stent position at the aneurysm orifice. Numerically we can observe this effect by moving the stent from the initial position.

Figures 3 and 4 show that the variation of the stent strut position influences the effective porosity, which is defined as the metal-free area divided by the orifice area. The positioning error is defined as the distance from the initial position divided by the characteristic cyclic length of the stent. This effect is most remarkable in the small-necked aneurysm with the large-pore stent and in this case the variation of the flow reduction by the positioning effect is large. These figures show that the stent property depending on the aneurysm size can be described quantitatively by measuring the effective porosity using the stent positioning effect. Figures 5 and 6 show the variation in the velocity reduction depending on the positioning error. Here, the velocity reduction  $V_r$  is defined by computing a coefficient comparing the nonstented and the stented cases:  $V_r = (U_{ns} - U_s) / U_{ns}$  where  $U_{ns}$  and  $U_s$  are the averages of the velocity  $u = \sqrt{u_x^2(\mathbf{r}) + u_y^2(\mathbf{r})}$  over all lattice sites  $\mathbf{r}(x, y)$  in the cavity, for nonstented and stented flows, respectively. The  $x$  axis and the  $y$  axis are defined in Fig. 1. The average velocity cannot reveal the mechanism of the localized stagnation within the aneurysm, which is thought

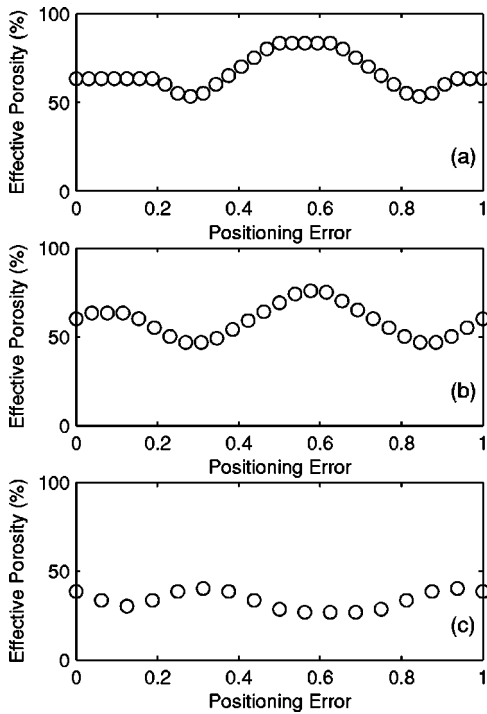


FIG. 3. Variation in the effective porosity at the small-necked aneurysm orifice when the stent is shifted along the cavity orifice successively by five cells: the positioning error is defined as the distance from the initial position divided by the characteristic cyclic length of the stent. (a) The case of stent A, (b) the case of stent B, and (c) the case of stent C. The variation in porosity is large at the small-necked aneurysm compared with the case at the large-necked aneurysm.

of as a cause of thrombotic occlusion, however, we focus on the total flow reduction property by stent implantation here. Figures 3–6 show the possibility that we can predict the velocity reduction effect by stent implantation in advance using the stent positioning effect. We can also characterize the velocity reduction property of the stent on each aneurysm using a Fourier series

$$V_r(\varepsilon) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos 2n\pi\varepsilon + \sum_{n=1}^{\infty} b_n \sin 2n\pi\varepsilon, \quad (1)$$

where  $\varepsilon$  is the positioning error. The coefficients  $a_0$ ,  $a_n$ , and  $b_n$  are stent-dependent parameters that provide a way to summarize the efficiency of a stent and, thus, give useful guidelines to design them.

Figures 5 and 6 also show the following features. Generally small-necked aneurysms have large velocity reduction error based on the stent position (see Fig. 5). This is remarkable in the large-pore stent (stent A). We think that this is due to the insufficient number of stent strut at the aneurysm orifice. This tendency is improved by using the small-pore stent (stent B or C). On the contrary, Fig. 6 shows that in the large-necked aneurysm the large-pore stent does not cause serious variations in flow reduction depending on the number of struts at the orifice, however this is sometimes the case for the small-pore stent. We think that this phenomenon is

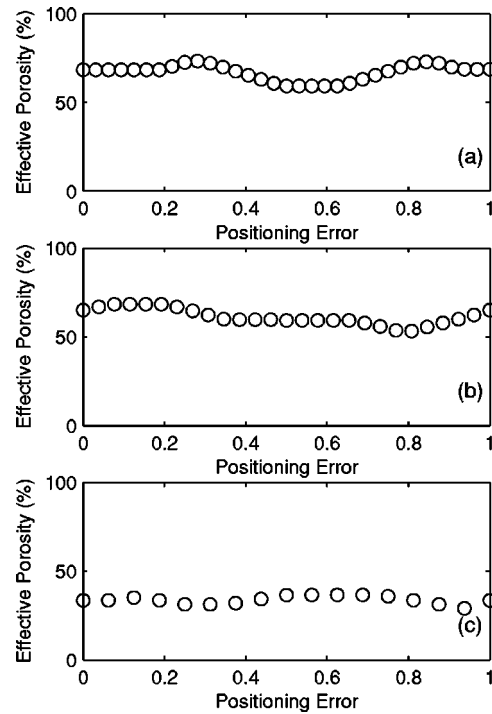


FIG. 4. Variation in the effective porosity at the large-necked aneurysm orifice when the stent is shifted along the cavity orifice successively by five cells. (a) The case of stent A, (b) the case of stent B, and (c) the case of stent C. The variation in porosity is not so large compared with the case of the small-necked aneurysm.

mainly caused by the strong orifice effect in the large-necked aneurysm. Compared with Fig. 7(c), Fig. 8(c) shows that the proportion maximum  $u_y$  of mean  $u$  at the aneurysm orifice is relatively large. This is due to a kind of orifice effect caused by the small pore in the stent and the relatively large velocity at the large-necked aneurysm orifice. These results show that the orifice effect cannot be neglected in determining the velocity reduction in large-necked aneurysms.

Flow reduction in an aneurysm by stent implantation is a complex phenomenon. Figures 3–6 show that there is no significant correlation between effective porosity and velocity reduction properties. These observations show that not only effective porosity but also strut distribution plays an important role in determination of the flow reduction effect. This is because it can be thought that the flow reduction mechanism is sensitive to the strut position due to the non-uniform velocity distribution at the aneurysm orifice, therefore the position of the strut at the aneurysm orifice is very important. Moreover the strut sometimes makes a narrow gap at the distal orifice. This also influences the orifice effect. These assumptions may explain the reason for the discontinuity in the variation in these figures. These phenomena, which are considered to be caused by the stent positioning effect or the orifice effect, have been observed experimentally [13]. We think that the stent positioning effect makes it difficult to analyze clinical observations, because it is difficult to know the exact stent position at the aneurysm orifice, therefore these numerical analyses will be helpful in understanding the clinical stent effect.

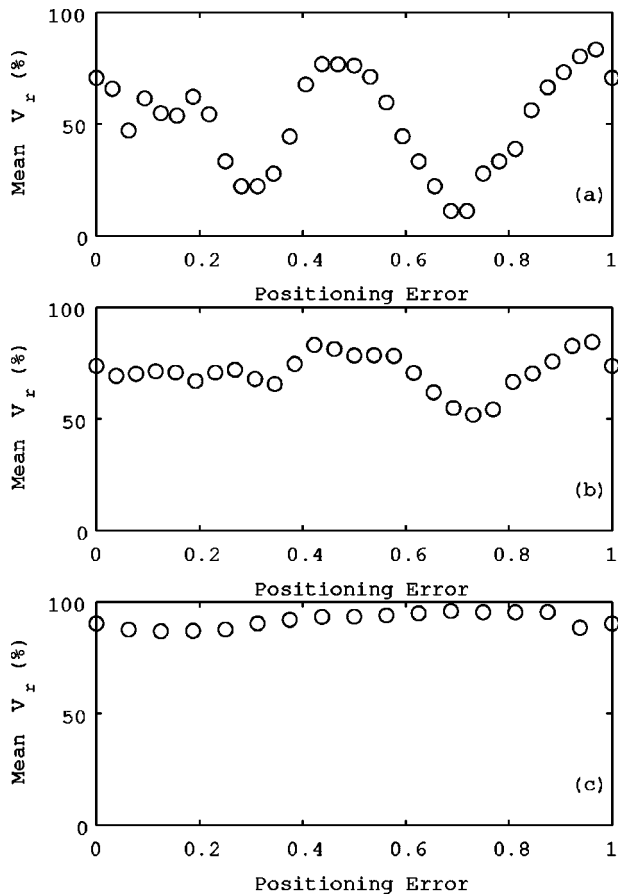


FIG. 5. Mean velocity reduction plots at the small-necked aneurysm orifice as a function of the positioning error. (a) The case of stent A, (b) the case of stent B, and (c) the case of stent C.

In conclusion, we have provided a useful guideline, the stent positioning measurement, in order to analyze the flow reduction effect in a cerebral artery with an aneurysm by stent implantation. The following four main analyses were obtained using this stent positioning measurement. First, the stent positioning effect cannot be neglected and therefore we can characterize the flow reduction property of a stent for each aneurysm by a Fourier series [Eq. (1)] utilizing this positioning effect. Second, a large-pore stent shows large fluctuation in the flow reduction for the small-necked aneurysm. We think that this is due to the small number of struts at the aneurysm orifice. This phenomenon depends mainly on the pore size (or effective porosity) of a stent. We guess that we can find the critical value of a combination of both the pore size and the aneurysm orifice size for characterizing these fluctuations. Third, a kind of orifice effect of the stent with small pores in the large-necked aneurysm disturbs the velocity reduction. This phenomenon depends mainly on the geometrical pore size, however, sometimes the strut position can cause another orifice effect because even a large-pore stent can make a narrow gap at the distal part of the aneurysm orifice. This implies that exact strut position at the aneurysm orifice plays an important role in flow reduction. Finally, the strut position (or strut distribution) at the aneurysm orifice must play another important role in the velocity reduction due to the nonuniform velocity distribution at the

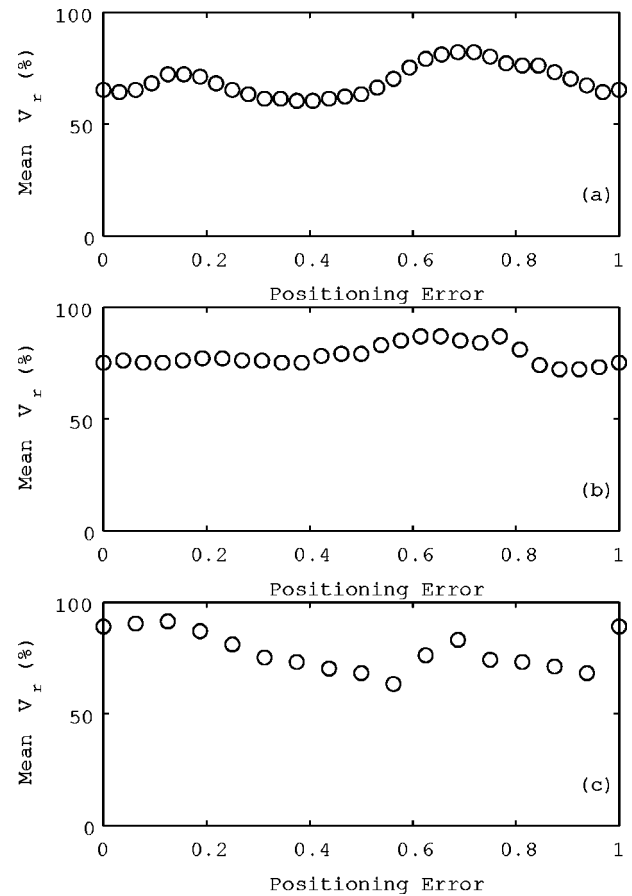


FIG. 6. Mean velocity reduction plots at the large-necked aneurysm orifice as a function of the positioning error. (a) The case of stent A, (b) the case of stent B, and (c) the case of stent C.

aneurysm orifice induced by the parent vessel flow. The effect of the flow reduction is sensitive to the strut distribution due to this nonuniformity. This may be one of the reasons for the existence of the discontinuity in the variation, which is observed in several figures.

We think that our analysis is effective in the three-dimensional case, too. As we mentioned before, the *in vitro* experiment in Ref. [8] shows a similar flow structure as our two-dimensional numerical simulation in Ref. [14]. In both cases, we can observe that the stent prevents the vortex in the aneurysm from being driven directly by the parent vessel flow. We provided the two-dimensional analysis of the mechanism of this flow pattern formation in detail in Ref. [14]. As for the stent design, the three-dimensional stent has more complex geometrical structure, however, the pore distribution and pore size should play the same important roles in this case, too. We think that the stent design parameters (such as the pore size, its distribution, the strut size, and its distribution) are very important factors in determining the flow structure. If the characteristic cyclic length of the stent is small (for example, a uniform fine-meshed stent), the stent positioning effect is not so remarkable as in our stent models used here. On the other hand, if the characteristic cyclic length of the stent is large (for example, a large-meshed stent), the flow reduction property is subject to the stent positioning effect and the strut distribution (or the pore distri-

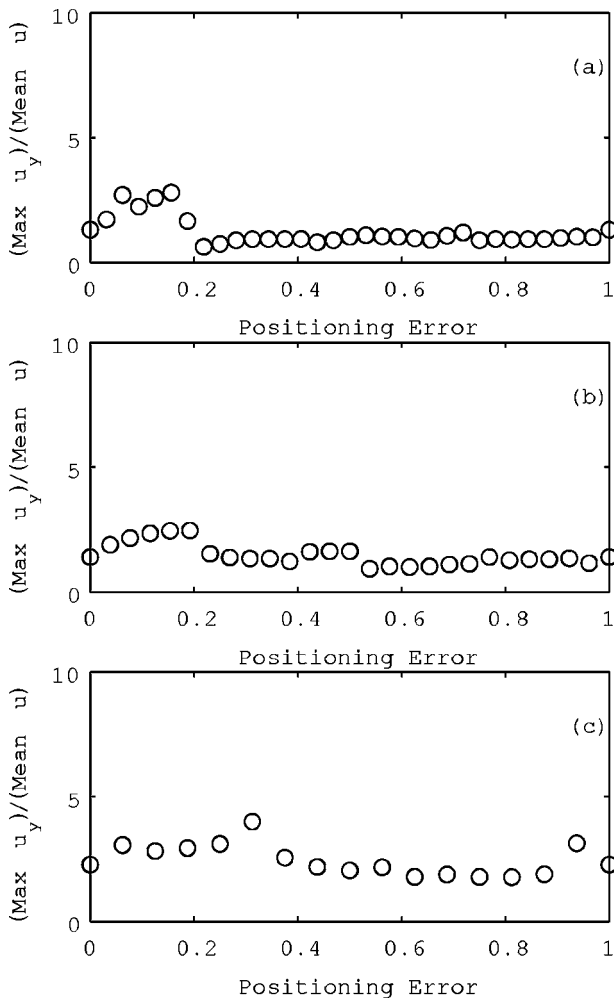


FIG. 7. The proportion maximum  $u_y$  of mean  $u$  plots at the small-necked aneurysm orifice with different positionings of the stent. (a) The case of stent A, (b) the case of stent B, and (c) the case of stent C.

bution) at the aneurysm orifice is also important for the flow pattern formation due to the nonuniform property of the velocity distribution at the aneurysm orifice. We expect that these design parameters can be classified using Eq. (1).

For future works, we think that we can propose some critical values of the combination of both stent parameters (such as the pore size or the strut size) and aneurysm parameters (such as the neck size or the dome size) on the flow reduction property by using the measurement of the positioning effect. Moreover the positioning effect will be useful to investigate the further flow structure for understanding the flow reduction mechanism by stent implantation. On a clinical note, it is difficult to confirm the stent positioning effect

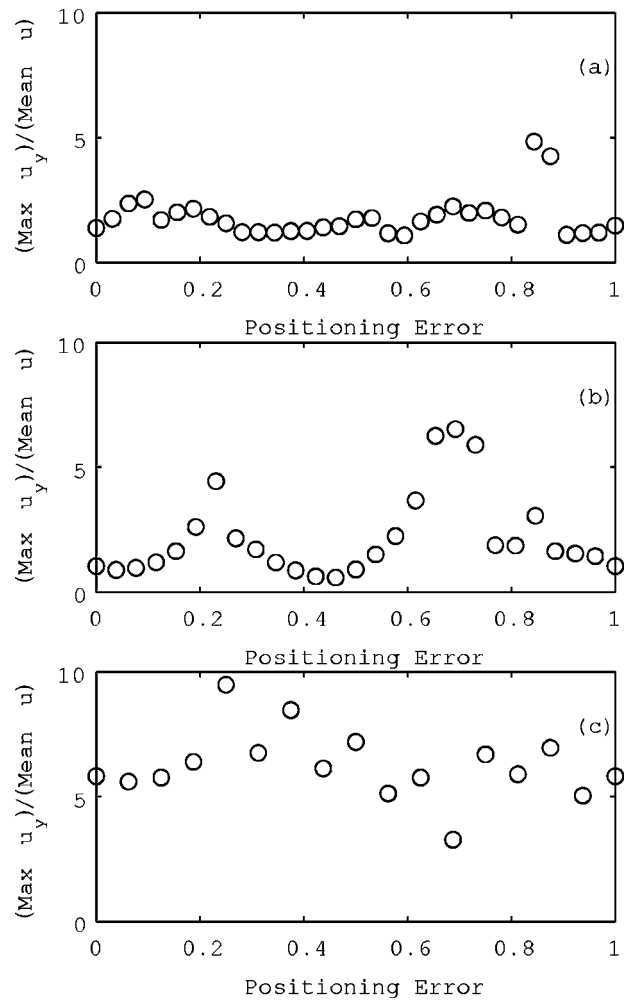


FIG. 8. The proportion maximum  $u_y$  of mean  $u$  plots at the large-necked aneurysm orifice with different positionings of the stent. (a) The case of stent A, (b) the case of stent B, and (c) the case of stent C.

*in vivo*, because we cannot know the stent position at the aneurysm orifice in detail. However, we will be able to verify the existence of the stent positioning effect indirectly by observing the fact that the same stent shows different flow reduction properties in similar aneurysm situations. We think that this phenomenon, which is often observed in clinical operations, is caused by the large-pore stent, which is often used, because these stents are sensitive to the stent positioning effect. Our results offer an important framework to discuss the intricate properties of the stent.

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